## General Primal - Dual Pair:

Based on the standard form of primal, there are two important primal - dual pairs.

**Definition: 1** (Standard Primal Problem)

Maximize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ subject to the constraints:

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i; \quad i = 1, 2, \dots, m$$

$$x_j \geq 0; \quad j = 1, 2, \dots, n$$

**Dual Problem:**

Minimize $z^* = b_1w_1 + b_2w_2 + \dots + b_mw_n$ subject to the constraints:

$$a_{1j}w_1 + a_{2j}w_2 + \dots + a_{mj}w_m \geq c_j; \quad j = 1, 2, \dots, n$$

$w_i$ $(i = 1, 2, \dots, m)$ unrestricted

Note that $x_j$'s are the primal variables, $w_i$'s the dual variables and the other constants have their usual meanings

**Definition: 2** (Standard Primal Problem)

Minimize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ subject to the constraints:

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i; \quad i = 1, 2, \dots, m$$

$$x_j \geq 0; \quad j = 1, 2, \dots, n$$

**Dual Problem:**

Maximize $z^* = b_1w_1 + b_2w_2 + \dots + b_mw_m$ subject to the constraints;

$$a_{1j}w_1 + a_{2j}w_2 + a_{mj}w_m \leq c_j; \quad j = 1, 2, \dots, n$$

$w_i$ $(i = 1, 2, \dots, m)$ unrestricted.

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## Formulating a Dual Problem:

Various steps involved in the formulation of a primal - dual pair are;

Step 1: Put the given linear programming problem ~~The ... of these~~ into its standard form. Consider it as the primal problem

Step 2 Identify the ~~...~~ variables to be used in the dual problem. The number of these variables equals the number of constraints equations in the primal

Step 3 Write down the objective function of the dual, using the right-hand side constants of the primal constraints

If the primal problem is of maximization type, the dual will be a minimization problem and vice-versa.

Step 4 Making use of dual variable identified in step 2 write the constraints for the dual problem

a) If the primal is a maximization problem, the dual constraints must be all of '≥' type. If the primal is a minimization problem, the dual constraints must be all of '≤' type.

b) The column coefficients of the primal constraints become the row coefficients of the dual constraints.

c) The coefficients of the primal objective function become the right-hand side constants of the dual constraints

d) The dual variables are defined to be unrestricted in sign

<u>Step 5</u>: Using steps 3 and 4, write down the dual of the given L.P.P.

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501 Formulate the dual of the following linear programming Problem:

Maximize $z = 5x_1 + 3x_2$ subject to the constraints

$3x_1 + 5x_2 \le 15$, $5x_1 + 2x_2 \le 10$, $x_1 \ge 0$ and $x_2 \ge 0$

<u>Soln</u>:

<u>Standard Primal</u>

Maximize $z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

subject to constraints,

$$3x_1 + 5x_2 + s_1 = 15$$

$$5x_1 + 2x_2 + s_2 = 10$$

$$x_1, x_2, s_1, s_2 \ge 0$$

<u>Dual</u>:

Minimize $z = 15w_1 + 10w_2$

Subject to constraints,

$$3w_1 + 5w_2 \ge 5$$

$$5w_1 + 2w_2 \ge 3$$

$$w_1 + 0w_2 \ge 0$$

$$0w_1 + w_2 \ge 0$$

Minimize $z = 15w_1 + 10w_2$

subject to constraints,

$$3w_1 + 5w_2 \ge 5$$

$5w_1 + 2w_2 \geq 3$

$w_1 \geq 0$

$w_2 \geq 0$ $\quad w_1, w_2$ unrestricted.

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5.02 Write the dual of the L.P.P

Minimize $z = 4x_1 + 6x_2 + 18x_3$

subject to constraints,

$x_1 + 3x_2 \geq 3$ , $x_2 + 2x_3 \geq 5$ and $x_j \geq 0$. $(j = 1, 2, 3)$

Soln:

Standard Primal:

Minimize $z = 4x_1 + 6x_2 + 18x_3 + 0s_1 + 0s_2$

Subject to constraints.

$x_1 + 3x_2 - s_1 = 3$

$0x_1 + x_2 + 2x_3 - s_2 = 5$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Dual:

Maximize $z^* = 3w_1 + 5w_2$

Subject to the constraints

$w_1 + 0w_2 \leq 4$

$3w_1 + w_2 \leq 6$

$0w_1 + 2w_2 \leq 18$

$-w_1 + 0w_2 \leq 0$

$0w_1 - w_2 \leq 0$

$w_1$ and $w_2$ unrestricted.

Maximize $z^* = 3w_1 + 5w_2$

Subject to the constraints

$w_1 \leq 4$

$3w_1 + w_2 \le 6$

$2w_2 \le 18$

$w_1 \ge 0$

$w_2 \ge 0$ $\qquad$ $w_1, w_2$ unrestricted.

---

50) Write the dual of the following linear programming problem.

Minimize $z = 3x_1 - 2x_2 + 4x_3$

Subject to constraints,

$3x_1 + 5x_2 + 4x_3 \ge 7,$

$6x_1 + x_2 + 3x_3 \ge 4,$

$7x_1 - 2x_2 - x_3 \le 10,$

$x_1 - 2x_2 + 5x_3 \ge 3,$

$4x_1 + 7x_2 - 2x_3 \ge 2$ $\qquad$ $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.$

Soln:

Standard Primal:

Minimize $z = 3x_1 - 2x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 0s_1$

Subject to constraints,

$3x_1 + 5x_2 + 4x_3 - S_1 = 7$

$6x_1 + x_2 + 3x_3 - S_2 = 4$

$7x_1 - 2x_2 - x_3 + s_1 = 10$

$x_1 - 2x_2 + 5x_3 - S_3 = 3$

$4x_1 + 7x_2 - 2x_3 - S_4 = 2$

$\therefore x_1, x_2, x_3, S_1, S_2, S_3, S_4, s_1 \ge 0$

Dual:

Maximize $z^* = 7w_1 + 4w_2 + 10w_3 + 3w_4 + 2w_5$

Subject to constraints,

$$3w_1 + 6w_2 + 7w_3 + w_4 + 4w_5 \le 3$$

$$5w_1 + w_2 - 2w_3 - 2w_4 + 7w_5 \le -2$$

$$4w_1 + 3w_2 - w_3 + 5w_4 - 2w_5 \le 4$$

$$-w_1 + 0w_2 + 0w_3 + 0w_4 + 0w_5 \le 0$$

$$0w_1 - w_2 + 0w_3 + 0w_4 + 0w_5 \le 0$$

$$0w_1 + 0w_2 + w_3 + 0w_4 + 0w_5 \le 0$$

$$0w_1 + 0w_2 + 0w_3 - w_4 + 0w_5 \le 0$$

$$0w_1 + 0w_2 + 0w_3 + 0w_4 - w_5 \le 0$$

$-w_1 \le 0$ , $-w_2 \le 0$ , $w_3 \le 0$ , $-w_4 \le 0$ , $-w_5 \le 0$

$\therefore w_1 \ge 0$ , $\therefore w_2 \ge 0$ , $\therefore w_3 \ge 0$ , $\therefore w_4 \ge 0$ , $\therefore w_5 \ge 0$

$w_1, w_2, w_3, w_4, w_5$ are unrestricted

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506 Obtain the dual of the following L.P.P :

Maximize $z = 2x_1 + 3x_2 + x_3$

Subject to constraints

$$4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \ge 0$$

Soln:

Standard Primal:

Maximize $z = 2x_1 + 3x_2 + x_3$

Subject to the constraints,

$$4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \ge 0$$

Dual:

Minimize $z^* = 6w_1 + 4w_2$

subject to constraints,

$$4\omega_1 + \omega_2 \geq 2$$

$$3\omega_1 + 2\omega_2 \geq 3$$

$$\omega_1 + 5\omega_2 \geq 1$$

$\omega_1$ and $\omega_2$ are unrestricted.

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## Duality Theorems:

Theorem: 5-1

The dual of the dual is the primal.

**Proof:**

**Primal:**

Let the primal L.P.P be,

Maximize $f(x) = cx$

subject to the constraints:

$$Ax \leq b$$

$$x \geq 0$$

**Dual:**

By definition its is given by

Minimize $f(\omega) = \omega = b^T\omega$

subject to the constraints

$$A^T\omega \geq c^T$$

$$\omega \geq 0$$

**Cannon:**

The cannonical form of the duel

Maximize $f(\omega) = -b^T\omega$

subject to the constraints

$$-A^T\omega \leq -c^T$$

$w \geq 0$

Hence dual of the dual is

Minimize $h(y) = (-c^T)^T y$

$$(-A^T)^T y \geq (-b^T)^T$$

$$\therefore y \geq 0$$

Maximize $h(y) = -(-cy)$

$$-Ay \geq -b$$

$$y \geq 0$$

ie) Maximize $h(y) = cy$ subject to,

$$Ay \leq b$$

$$y \geq 0$$

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503 Obtain the dual problem of the following primal problem:

Minimize $z = x_1 - 3x_2 - 2x_3$ subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7,$$

$$2x_1 - 4x_2 \geq 12,$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

$x_1, x_2 \geq 0$ and $x_3$ is unrestricted.

Soln:

Standard Primal:

Minimize $z = x_1 - 3x_2 - 2(x_3' - x_3'') + 0s_1$

$$= x_1 - 3x_2 - 2x_3' + 2x_3'' + 0s_1 + 0s_1$$

Subject to the constraints

$$3x_1 - x_2 + 2x_3' - 2x_3'' + s_1 = 7$$

$$2x_1 - 4x_2 - s_1 = 12$$

$$-4x_1 + 3x_2 + 8x_3' - 8x_3'' = 0$$

$x_1, x_2, x_3', x_3'', s_1, s_1 \geq 0$

**Dual:**

Maximize $z^* = 7w_1 + 12w_2 + 10w_3$

subject to the constraints,

$3w_1 + 2w_2 - 4w_3 \leq 1$

$-w_1 - 4w_2 + 3w_3 \leq -3$

$\therefore -(w_1 + 4w_2 - 3w_3) \leq -3$

$(w_1 + 4w_2 - 3w_3) \geq 3$

$$\left.\begin{array}{l} 2w_1 + 8w_3 \leq -2 \\ -2w_1 - 8w_3 \leq 2 \end{array}\right\} -2w_1 - 8w_3 = 2$$

$w_1 + 0w_2 \leq 0 \Rightarrow w_1 \leq 0$

$0w_1 - w_2 \leq 0 \Rightarrow -w_2 \leq 0$

$w_2 \geq 0$

$w_1, w_2, w_3$ are unrestricted.

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504. Obtain the dual problem of the following L.P.P

Maximize $f(x) = 2x_1 + 5x_2 + 6x_3$

subject to constraints;

$5x_1 + 6x_2 - x_3 \leq 3$,

$-2x_1 + x_2 + 4x_3 \leq 4$,

$x_1 - 5x_2 + 3x_3 \leq 1$

$-3_1 - 3x_2 + 7x_3 \leq 6$

$x_1, x_2, x_3 \geq 0.$

**Soln:**

**Standard Primal:**

Maximize $f(x) = 2x_1 + 5x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

subject to the constraints

$$5x_1 + 6x_2 - x_3 + s_1 = 3$$

$$-2x_1 + x_2 + 4x_3 + s_2 = 4$$

$$x_1 - 5x_2 + 3x_3 + s_3 = 1$$

$$-3x_1 - 3x_2 + 7x_3 + s_4 = 6$$

$$x_1, x_2, x_3, s_1, s_2, s_3, s_4 \geq 0$$

Dual:

Minimize $f(x)^* = 3w_1 + 4w_2 + w_3 + 6w_4$

subject to the constraints

$$5w_1 - 2w_2 + w_3 - 3w_4 \geq 3$$

$$6w_1 + w_2 - 5w_3 - 3w_4 \geq 4$$

$$-w_1 + 4w_2 + 3w_3 + 7w_4 \geq 1$$

$$w_1 + 0w_2 + 0w_3 + 0w_4 \geq 0 \Rightarrow w_1 \geq 0$$

$$0w_1 + w_2 + 0w_3 + 0w_4 \geq 0 \Rightarrow w_2 \geq 0$$

$$0w_1 + 0w_2 + w_3 + 0w_4 \geq 0 \Rightarrow w_3 \geq 0$$

$$0w_1 + 0w_2 + 0w_3 + w_4 \geq 0 \Rightarrow w_4 \geq 0$$

$w_1, w_2, w_3, w_4$ are restricted.

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505 Find the dual of the following L.P.P:

Maximize $z = 2x_1 + x_2$ subject to the constraints

$$x_1 + 5x_2 \leq 10$$

$$x_1 + 3x_2 \geq 6$$

$2x_1 + 2x_2 \leq 8$; $x_2 \geq 0$ and $x_1$ unrestricted.

Soln:

Standard Primal:

Maximize $z = 2(x_1 - x_1'') + x_2 + 0s_1 - 0s_1 + 0s_2$

$= 2x_1 - 2x_1'' + x_2 + 0s_1 - 0s_1 + 0s_2$

subject to the constraints.

$$x_1' - x_1'' + 5x_2 + s_1 = 10$$

$$x_1' - x_1'' + 3x_2 - s_1 = 6$$

$$2x_1' - 2x_1'' + 2x_2 + s_2 = 8$$

Dual:

Minimize $z^* = 10w_1 + 6w_2 + 8w_3$

subject to the constraints,

$$\left.\begin{aligned} w_1 + w_2 + 2w_3 &\geq 2 \\ -w_1 - w_2 - 2w_3 &\geq -2 \end{aligned}\right\} -w_1 - w_2 - 2w_3 = 2$$

$$5w_1 + 3w_2 + 2w_3 \geq 1$$

$$w_1 - 0w_2 + 0w_3 \geq 0 \Rightarrow w_1 \geq 0$$

$$0w_1 - w_2 + 0w_3 \geq 0 \Rightarrow -w_2 \geq 0 \Rightarrow w_2 \leq 0$$

$$0w_1 - 0w_2 + w_3 \geq 0 \Rightarrow w_3 \geq 0$$

$w_1, w_2, w_3$ are unrestricted.

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507 Obtain the dual problem of the following L.P.P

Maximize $z = x_1 - 2x_2 + 3x_3$ subject to the constraints

$$-2x_1 + x_2 + 3x_3 = 2,$$

$$2x_1 + 3x_2 + 4x_3 = 1; \quad x_1, x_2, x_3 \geq 0$$

Soln:

Dual:

Minimum $z^* = 2w_1 + w_2$

Subject to the constraints,

$$-2w_1 + 2w_2 \geq 1$$

$$w_1 + 3w_2 \geq -2$$

$$3w_1 + 4w_2 \geq 3$$

$w_1$ and $w_2$ are unrestricted.

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510 Minimize $z = x_1 + x_2 + x_3$

Subject to the constraints,

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \le 3$$

$$2x_2 - x_3 \ge 4$$

$x_1, x_2 \ge 0$ and $x_3$ unrestricted.

Soln:

Standard Primal:

Minimize $z = x_1 + x_2 + x_3' - x_3'' + 0s_1 + 0S_1$

Subject to the constraints,

$$x_1 - 3x_2 + 4x_3' - 4x_3'' = 5$$

$$x_1 - 2x_2 + s_1 = 3$$

$$2x_2 - x_3' + x_3'' - S_1 = 4$$

$$x_1, x_2, x_3', x_3'', s_1, S_1 \ge 0$$

Dual:

Maximum $z^* = 5w_1 + 3w_2 + 4w_3$

Subject to the constraints

$$w_1 + w_2 + 0w_3 \le 1$$

$$-3w_1 - 2w_2 + 2w_3 \le 1$$

$$\left.\begin{aligned} 4w_1 + 0w_2 - w_3 &\le 1 \\ -4w_1 + 0w_2 + w_3 &\le -1 \end{aligned}\right\} -4w_1 + 0w_2 + w_3 = 1$$

$$0w_1 + 0w_2 + 0w_3 \le 0$$

$$0w_1 + w_2 + 0w_3 \le 0 \Rightarrow w_2 \le 0$$

$$0w_1 + 0w_2 - w_3 \le 0 \Rightarrow w_3 \ge 0$$

$w_1, w_2, w_3$ are unrestricted.

511. Give the dual of the following L.P.P.

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Minimize $z = 2x_1 + 3x_2 + 4x_3$ subject to the constraints

$2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 5$ $\quad x_1, x_2 \geq 0$ and $x_3$ is unrestricted

soln:

Standard Primal:

Minimize $z = 2x_1 + 3x_2 + 4x_3' - 4x_3'' + 0s_1 + 0s_1$

Subject to the constraints

$2x_1 + 3x_2 + 5x_3' - 5x_3'' - s_1 = 2$

$3x_1 + x_2 + 7x_3' - 7x_3'' = 3$

$x_1 + 4x_2 + 6x_3' - 6x_3'' + s_1 = 5$

$x_1, x_2, x_3', x_3'', s_1, s_1 \geq 0$

Dual:

Maximize $z^* = 2w_1 + 3w_2 + 5w_3$

Subject to the constraints

$2w_1 + 3w_2 + w_3 \leq 2$

$3w_1 + w_2 + 4w_2 \leq 3$

$$\left.\begin{array}{l} 5w_1 + 7w_2 + 6w_3 \leq 4 \\ -5w_1 - 7w_2 - 6w_3 \leq -4 \end{array}\right\} -5w_1 - 7w_2 - 6w_3 = 4$$

$-w_1 + 0w_2 + 0w_3 \leq 0 \Rightarrow -w_1 \leq 0 \Rightarrow w_1 \geq 0$

$-0w_1 + 0w_2 + w_3 \leq 0 \Rightarrow w_3 \leq 0$

$\therefore w_1, w_2, w_3$ are unrestricted

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512 Maximize $z = 6x_1 + 6x_2 + x_3 + 7x_4 + 5x_5$

subject to the constraints

$3x_1 + 7x_2 + 8x_3 + 5x_4 + x_5 = 2$

$2x_1 + x_2 + 3x_3 + 2x_4 + 9x_5 = 6$

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$x_1, x_2, x_3, x_4 \geq 0$ , $x_5$ unrestricted

Soln:

Standard Primal:

Maximize $z = 6x_1 + 6x_2 + x_3 + 7x_4 + 5x_5' - 5x_5''$

Subject to the constraints

$3x_1 + 7x_2 + 8x_3 + 5x_4 + x_5' - x_5'' = 2$

$2x_1 + x_2 + 3x_3 + 2x_4 + 9x_5' - 9x_5'' = 6$

$x_1, x_2, x_3, x_4, x_5', x_5'' \geq 0$

Dual:

Min $z^* = 2w_1 + 6w_2$

Subject to the constraints

$3w_1 + 2w_2 \geq 6$

$7w_1 + w_2 \geq 6$

$8w_1 + 3w_2 \geq 1$

$5w_1 + 2w_2 \geq 7$

$$\left.\begin{array}{l} w_1 + 9w_2 \geq 5 \\ -w_1 - 9w_2 \geq -5 \end{array}\right\} -w_1 - 9w_2 = 5$$

$w_1, w_2$ are unrestricted.

Theorem: 5.2 (Weak Duality Theorem)

Let $X_0$ be a feasible solution to the primal problem Maximize $f(x) = cx$ subject to $Ax \leq b$, $x \geq 0$ where $x^T$ and $c \in R^n$, $b^T \in R^m$ and $A$ is an $m \times n$ real matrix. If $w_0$ be a feasible solution to the dual of the primal, namely,

Minimize $g(w) = b^T w$ subject to, $A^T w \geq c^T$, $w \geq 0$, where $w^T \in R^m$, then $cX_0 \leq b^T w_0$

Proof:

Since $x_0$ is a feasible solution to the primal problem, we have

$$Ax_0 \le b \quad , x_0 \ge 0 \rightarrow ①$$

Since $w_0$ is a feasible solution to the dual problem we have,

$$A^T w_0 \ge c^T \quad , w_0 \ge 0 \rightarrow ②$$

Now, Taking transpose on both sides in equation ② we have,

$$c \le A w_0^T$$

$$cx_0 \le A w_0^T x_0$$

$$cx_0 \le w_0^T (Ax_0)$$

$$cx_0 \le w_0^T b \quad \text{(by ①)}$$

Taking transpose on both sides

$$(cx_0)^T \le w_0 b^T \quad (\because cX_0 \text{ is a real number}$$

$$cX_0 \le b^T w_0 \quad (cx_0)^T = cx_0 )$$

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Theorem: 5.3

Let $x_0$ be a feasible solution to the primal problem.

Maximize $f(x) = cx$ subject to $Ax \le b$, $x \ge 0$

and $w_0$ be a feasible solution to its dual

Minimize $g(w) = b^T w$ subject to : $A^T w \ge c^T$, $w \ge 0$

where $x^T$ and $c \in R^n$, $w^T$ and $b^T \in R^m$ and $A$ is an $m \times n$ real matrix

If $cx_0 = b^T w_0$ then both $x_0$ and $w_0$ are optimum solutions to the primal and dual respectively.

**Proof**:

Let $x_0^*$ be any feasible soln to the primal Problem.

By theorem, 5.2, $cx_0^* \leq b^T w_0$

$\therefore cx_0^* \leq cx_0 \quad (cx_0 = b^T w_0)$

$\therefore x_0$ is an optimal solution to its primal.

Now, let $w_0^*$ be any feasible solution to the dual problem.

By theorem 5.2, $cx_0 \leq b^T w_0^*$

$b^T w_0 \leq b^T w_0^* \quad [\because cx_0 = b^T w_0]$

$\therefore w_0$ is an optimal solution to the dual problem

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**Duality and simplex method**:

The fundamental theorem of Duality suggests than an optimum solution to the associated dual can be obtained from that of its primal and vice versa.

If primal is a maximaization problem, then following are the set of rules

**Rule: 1**

Corresponding net evaluation of the starting primal variables = difference between the left and right sides of the dual constraints associated with the starting primal variable.

**Rule: 2**

Negative of the corresponding net evaluations of the starting dual variable = Difference between the left and right sides of the primal constraints associated with dual starting variables.

Rule: ③

If the primal (dual) problem is unbounded then the dual (Primal) Problem does not have any feasible solution.

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5.13 Use duality to solve the following L.P.P

Max $z = 2x_1 + x_2$ subject to constraints

$x_1 + 2x_2 \leq 10$, $x_1 + x_2 \leq 6$, $x_1 - x_2 \leq 2$,

$x_1 - 2x_2 \leq 1$ ; $x_1, x_2 \geq 0$

Soln:

Dual Problem

Minimize $w = 10w_1 + 6w_2 + 2w_3 + w_4$

subject to constraints

$w_1 + w_2 + w_3 + w_4 \geq 2$

$2w_1 + w_2 - w_3 - 2w_4 \geq 1$

$w_1, w_2, w_3, w_4 \geq 0$

Standard form:

Maximum $w = -10w_1 - 6w_2 - 2w_3 - w_4 + 0s_1 + 0s_2 - Ma_1 - Ma_2$

subject to constraints

$w_1 + w_2 + w_3 + w_4 - s_1 + a_1 = 2$

$2w_1 + w_2 - w_3 - 2w_4 - s_2 + a_2 = 1$

$w_1, w_2, w_3, w_4, s_1, s_2, a_1, a_2 \geq 0$

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| | | | -10 | -6 | -2 | -1 | 0 | 0 | -M | -M | ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | B | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $s_1$ | $s_2$ | $a_1$ | $a_2$ | |
| -M | $a_1$ | 2 | 1 | 1 | 1 | 1 | -1 | 0 | 1 | 0 | $2/1 = 2$ |
| -M | $a_2$ | 1 | 2 | 1 | -1 | -2 | 0 | -1 | 0 | 1 | $1/2 = 0.5$ |
| | [illegible] | $z_j$ -3M | -3M | -2M | 0 | M | M | M | -M | -M | |
| | | $z_j - c_j$ | -3M+10 | -2M+6 | 2 | M+1 | M | M | 0 | 0 | |
| -M | $a_1$ | $3/2$ | 0 | $1/2$ | $3/2$ | 2 | -1 | $1/2$ | 1 | $-1/2$ | $3/2 \times 1/2 = 3/4$ |
| -10 | $w_1$ | $1/2$ | 1 | $1/2$ | $-1/2$ | -1 | [illegible] | -[illegible]/2 | 0 | $1/2$ | $1/2 \times -1 = -1/2$ |
| | | $z_j$ | -10 | $\frac{-M-10}{2}$ | $\frac{-3M+10}{2}$ | -2M+10 | M | $\frac{-M+10}{2}$ | -M | $-\frac{M-5}{2}$ | |
| | | $z_j - c_j$ | 0 | $\frac{-M+2}{2}$ | $\frac{-3M+14}{2}$ | -2M+11 | M | $\frac{-M+10}{2}$ | 0 | $\frac{M-5}{2}$ | |
| -1 | $w_4$ | $3/4$ | 0 | $1/4$ | $3/4$ | 1 | $-1/2$ | $1/4$ | $1/2$ | $-1/4$ | $3/4 \times 4 = 3$ |
| -10 | $w_1$ | $5/4$ | 1 | $3/4$ | $1/4$ | 0 | $-1/2$ | $-1/4$ | $1/2$ | $1/4$ | $5/4 \times 4/3 = 16$ |
| | | $z_j$ | -10 | $\frac{-31}{4}$ | $\frac{-13}{4}$ | -1 | $11/2$ | $9/4$ | $-11/2$ | $-9/4$ | |
| | | $z_j - c_j$ | 0 | $-7/4$ | $-5/4$ | 0 | $11/2$ | $9/4$ | $-11/2+M$ | $-9/4+M$ | |
| -1 | $w_4$ | $1/3$ | $-4/12$ | 0 | $2/3$ | 1 | $-1/3$ | $1/3$ | $-1/3$ | $-1/3$ | $1/3 \times 3/2 = 1/2$ |
| -6 | $w_2$ | $5/3$ | $4/3$ | 1 | $1/3$ | 0 | $-2/3$ | $-1/3$ | $2/3$ | $1/3$ | $5/3 \times 3/1 = 5$ |
| | | $z_j$ | $\frac{-31}{3}$ | -6 | $-8/3$ | -1 | $13/3$ | $5/3$ | $-13/3$ | $-5/3$ | - |
| | | $z_j - c_j$ | $-1/3$ | 0 | $-2/3$ | 0 | $13/3$ | $5/3$ | $\frac{-13+3M}{3}$ | $-\frac{5+3M}{3}$ | |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| -2 | $w_3$ | 1/2 | -1/2 | 0 | $\overset{1}{\cancel{1/2}}$ | $\overset{3/2}{\cancel{[illegible]}}$ | -1/2 | 1/2 | 1/2 | -1/2 | | |
| -6 | $w_2$ | 3/2 | 3/2 | 1 | 0 | -1/2 | -1/2 | -1/2 | 1/2 | 1/2 | | |
| | | $z_j$ | -8 | -6 | -2 | 0 | 4 | 2 | -4 | -2 | | |
| | | $z_j - c_j$ | 2 | 0 | 0 | 1 | 4 | 2 | -4+m | -2+m | | |

Since all $z_j - c_j \geq 0$

Hence the optimum solution is

$w_1 = 0$ , $w_2 = 3/2$ , $w_3 = 1/2$ , $w_4 = 0$

starting dual variable : $a_1$ , $a_2$

$-(z_j - c_j)$ : $4-m$ $\quad 2-m$

Primal variable : $x_1$ $\quad x_2$

$x_1 = 4 \qquad x_2 = 2$

Maximum $z = 2(4) + 2 = 8 + 2 = 10$